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| Description: EGC_Black | **MATHEMATICS: SPECIALIST UNITS 1 & 2**  **INVESTIGATION 4**  **PART A** |

**MATHEMATICAL INDUCTION**

Date of Validation:

You will have time at home to work on Part A. I will check your take-home part on Friday 13th September and hand you fully worked solutions. Your validation will be on Tuesday 10th September. You WILL NOT be permitted to use Part A in Part B - the validation.

Mathematical Induction is a rigorous method of mathematical proof that adopts a consistent algebraic structure. The key part of the argument requires similar skills to the proving of trigonometric identities.

The principle of Mathematical Induction states:

“If a set of positive integers:

(a) contains the positive integer 1, and

(b) can be proved to contain the positive integer *k + 1* whenever it contains all positive integers 1, 2, 3, ..., *k*,

then the set contains all positive integers.”

The principle of Mathematical Induction is used to prove many results in algebra and calculus.

EXAMPLE 1

Prove:  for all positive integers *n*.

Step 1: Verify the statement is true when *n* = 1.

L.H.S. = 1

R.H.S. = 

= 1

⇒ Statement is true for *n* = 1.

Step 2: Assume the statement is true for *n* = *k*.

That is, 

Step 3: Prove the statement is true for *n* = *k* + 1

That is, prove 

L.H.S. =

 (From step 2)

=

=

=

=

= R.H.S.

⇒ The statement is true for *n* = *k* + 1 if it is true for *n* = *k*.

Step 4: As the statement is true for *n* = 1, it must be true for *n* = 2.

As the statement is true for *n* = 2, it must be true for *n* = 3 and so on.

Hence,  is true for all positive integers *n*.

EXAMPLE 2

Prove:  for all positive integer *n*.

Step 1: Verify the statement is true for *n* = 1.

L.H.S. = 2 × 1 − 1

= 1

R.H.S. = 

= 1

⇒ Statement is true for *n* = 1

Step 2: Assume the statement is true for *n* = k.

That is, 

Step 3: Prove the statement is true for *n* = *k* + 1.

That is, prove 

L.H.S. = 

=  (from step 2)

=

=

=

= R.H.S.

⇒ The statement is true for *n* = *k* + 1 if it true for *n* = *k*.

Step 4: As the statement is true for *n* = 1 it must be true for *n* = 2.

As the statement is true for *n* = 2 it must be true for *n* = 3 and so on.

Hence,  is true for all positive integers *n*.

EXAMPLE 3

Prove:  is divisible by 5 for all positive integers.

Step 1: Verify the statement is true when *n* = 1.



= 5

Divisible by 5.

⇒ Statement is true when *n* = 1.

Step 2: Assume true when *n* = *k*.

That is,  is divisible by 5.

Step 3: Prove the statement is true for n = k + 1

That is, prove that  is divisible by 5.



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Clearly  is divisible by 5 and

 is divisible by 5 because the factor  is divisible by 5.

Hence  is divisible by 5

⇒ The statement is true for *n* = *k* + 1 if it is true for *n* = *k*.

Step 4: As the statement is true for *n* = 1 it must be true for *n* = 2.

As the statement is true for *n* = 2 it must be true for *n* = 3 and so on.

Hence,  is divisible by 5 for all positive integers.

QUESTIONS TO ANSWER

Use the principle of Mathematical Induction to prove the following results are true for all positive integers *n*:

1. 

2. 

3. 

4. 

5.  is divisible by 6.

6. Prove for all natural numbers 

